Reinforcement Learning

Lecture 2 Classical Ideas

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Roadmap

- Introduction and overview

  motivation, bandits, big picture

- Classical ideas

  temporal difference methods, policy gradient, . . .

- Deep Reinforcement learning

  neural networks, DQN, DDPG, . . .

- Advanced techniques

  representation learning, stabilization, few-shot learning

- Applications

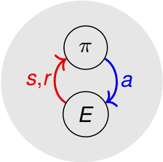
  AlphaGo, AlphaTensor, . . .

environment model

bandits, MDPs, POMDPs

learning target

model, value, policy



behavior policy

exploration vs exploitation

update rules

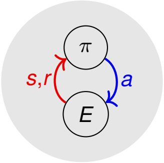
experience, loss

four dimensions

policy evaluation / prediction

$\pi, E/\text{interactions} \rightarrow V_\pi$

value iteration, linear system, Monte Carlo, ...



planning / control

$E \rightarrow \text{argmax}_\pi V_\pi$

value iteration, policy iteration, Monte Carlo, ...

reinforcement learning

interactions with $E \rightarrow \text{argmax}_\pi V_\pi$

Q-learning, SARSA, policy gradient, ...

π = *policy*, V_π = *policy value*, E = *environment*

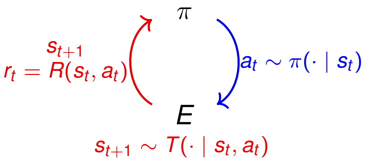
three problems

Markov Decision Processes (MDPs)

MABs are stateless, some problems have states $\Rightarrow$ MDP.
State: useful environment information for making decisions.

MDP $(p_0, S, A, T, R, \gamma)$

- initial state distribution p_0
- state space S
- action space A
- transition model $T(s' \mid s, a)$
- reward function $R(s, a)$
- discount factor $\gamma \in [0, 1)$



Objective: find an optimal (stochastic) policy $\pi : S \to \Delta_A$

$$\max_\pi V(\pi) := \mathbb{E}_{s \sim p_0} V_\pi(s) := \mathbb{E}_{s \sim p_0} \mathbb{E}(\sum_{t=0}^{\infty} \gamma^t r_t \mid s_0 = s, \pi).$$

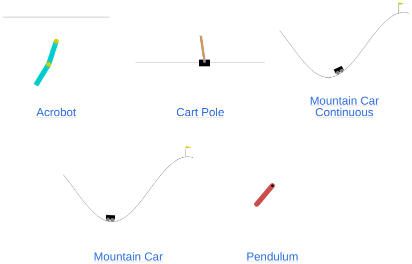
$V_\pi(s)$: value (expected total discounted reward) of π starting from s.
Δ_A: probability distributions on A.

Remarks. (a) often exists an optimal deterministic policy $\pi : S \to A$, so they are popular too.
(b) often interested in finite horizon problems too.

many publicly available benchmark environments

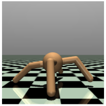
generally follow OpenAI's Gym API

Gym lives as Gymnasium now

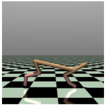


classic control

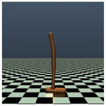
https://gymnasium.farama.org/environments/classic_control/



Ant

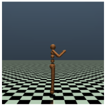


Half Cheetah

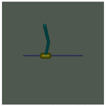


Hopper

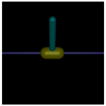
Humanoid Standup



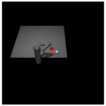
Humanoid



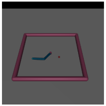
Inverted Double Pendulum



Inverted Pendulum



Pusher



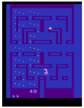
Reacher

MuJoCo

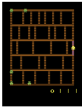
https://gymnasium.farama.org/environments/mujoco/

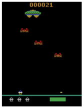

Adventure

Air Raid

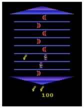


Alien


Amidar



Assault



Asterix

Asteroids


Atlantis

Atari

https://gymnasium.farama.org/environments/atari/

DACBench: a benchmark for Dynamic Algorithm Configuration

Gymnasium v0.26.3 Stars 27

A benchmark library for Dynamic Algorithm Configuration. Its focus is on reproducibility and comparability of different DAC methods as well as easy analysis of the optimization process.

flappy-bird-env

Gymnasium v0.28.1 Stars 19

Flappy Bird as a Farama Gymnasium environment.

flappy-bird-gymnasium: A Flappy Bird environment for Gymnasium

Gymnasium v0.27.1 Stars 41

A simple environment for single-agent reinforcement learning algorithms on a clone of Flappy Bird, the hugely popular arcade-style mobile game. Both state and pixel observation environments are available.

gym-cellular-automata: Cellular Automata environments

Gymnasium v0.28.1 Stars 32

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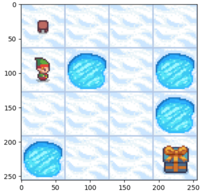
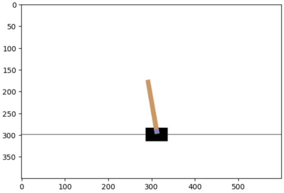
many 3rd party environments

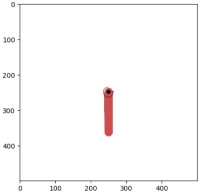
https://gymnasium.farama.org/environments/third_party_environments/



Frozen Lake
disc. S & A



Cartpole
cont. S & disc. A



Pendulum
cont. S & A

useful toy problems – good starting points

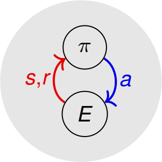
try me
https://colab.research.google.com/drive/1Qr19jg97Q4mRVrQ3yr3_mudEOD6IQCNr

Planning

policy evaluation / prediction

$\pi, E/\text{interactions} \;\rightarrow\; V_\pi$

value iteration, linear system, Monte Carlo, ...



planning / control

$E \;\rightarrow\; \text{argmax}_\pi\, V_\pi$

value iteration, policy iteration, Monte Carlo, ...

reinforcement learning

interactions with $E \rightarrow \text{argmax}_\pi\, V_\pi$

Q-learning, SARSA, policy gradient, ...

plan → RL

- a building block: learn a model, then plan
- a source of inspiration: sample-based approximation to exact operations in planning

- The optimal value function V^* satisfies the Bellman optimality equation

$$V^*(s) = \max_a \left(\sum_{s'} T(s' \mid s, a) \left(R(s, a) + \gamma V^*(s') \right) \right).$$

- Equivalently, V^* is the fixed point of the Bellman operator H:

$$V^* = H(V^*),$$

where the Bellman operator $H : \mathbf{R}^S \to \mathbf{R}^S$ is defined by

$$H(V)(s) \stackrel{def}{=} \max_a \left(\sum_{s'} T(s' \mid s, a) \left(R(s, a) + \gamma V(s') \right) \right),$$

- If we choose an arbitrary V_0, and $V_{t+1} = H(V_t)$, then V_t converges to V^* (proved using Banach fixed-point theorem).

Algorithm The Value Iteration algorithm for computing $\pi \approx \pi^*$

```
Initialize V_0                          ▷ often set to 0 if no good estimates available
for t = 1 to T do
    V_t ← H(V_{t-1})                    ▷ improve estimates using Bellman operator
    V ← V_t                             ▷ use V to remember most recent estimates
    Terminate if ‖V_t − V_{t-1}‖_∞ < ε
π(s) = argmax_a ( R(s,a) + γ Σ_{s'} T(s' | s,a) V(s') ).
```

Initialize V_0 ▷ often set to 0 if no good estimates available

for $t = 1$ to T **do**

$\quad V_t \leftarrow H(V_{t-1})$ ▷ improve estimates using Bellman operator

$\quad V \leftarrow V_t$ ▷ use V to remember most recent estimates

$\quad$ Terminate if $\| V_t - V_{t-1} \|_\infty < \epsilon$

$\pi(s) = \text{argmax}_a \left(R(s,a) + \gamma \sum_{s'} T(s' \mid s, a) V(s') \right)$.

computing π^* from V^* is expensive, and requires knowledge of R and T

- Often easier to work with *action-value function* $Q_\pi(s, a)$, defined as

$$Q_\pi(s, a) = \mathbb{E}(\sum_{t=0}^{\infty} \gamma^t r_t \mid s_0 = s, a_0 = a, \pi).$$

- Bellman equation for the optimal action value function Q^*:

$$Q^*(s, a) = \left(\sum_{s'} T(s' \mid s, a) \left(R(s, a) + \gamma \max_{a'} Q^*(s', a') \right) \right).$$

- Equivalently, Q^* is the fixed point of the Bellman operator H (*overloaded notation!*):

$$Q^* = H(Q^*),$$

where the Bellman operator $H : \mathbf{R}^{S \times A} \to \mathbf{R}^{S \times A}$ is defined by

$$H(Q)(s, a) \stackrel{def}{=} \left(\sum_{s'} T(s' \mid s, a) \left(R(s, a) + \gamma \max_{a'} Q(s', a') \right) \right),$$

- If we choose an arbitrary Q_0, and $Q_{t+1} = H(Q_t)$, then Q_t converges to Q^*.

Algorithm The Q-Iteration algorithm for computing $\pi \approx \pi^*$

Initialize Q_0 ▷ often set to 0 if no good estimates available
for $t = 1$ to T **do**
    $Q_t \leftarrow H(Q_{t-1})$ ▷ improve estimates using Bellman operator
    $Q \leftarrow Q_t$ ▷ use Q to remember most recent estimates
    Terminate if $\|Q_t - Q_{t-1}\|_\infty < \epsilon$
$\pi(s) = \operatorname{argmax}_a Q(s, a)$

π^* can be computed using Q alone, but $\operatorname{argmax}_a Q(s, a)$ can be hard

Q-learning

MDPs with finitely many states

- Q-learning (Watkins and Dayan, 1992) tries to directly estimate the optimal Q-function by solving the Bellman optimality equation

$$Q^*(s,a) = \sum_{s'} T(s' \mid s,a)(R(s,a) + \max_{a'} Q^*(s',a'))$$

  Key idea: replace expectation wrt s' using a sampled transition.

- If we experience a transition (s, a, s', r), then we can use it to perform an update

$$Q(s,a) \leftarrow Q(s,a) + \alpha\underbrace{(\overbrace{r + \gamma \max_{a'} Q(s',a')}^{\text{TD target}} - Q(s,a))}_{\text{TD}},$$

  where $\alpha > 0$ is the learning rate, s is the current state, and s' and r are the next state and the reward obtained after executing a.

- Think of α as a level of trust on the sampled transition.

Algorithm Tabular Q-learning

Initialise the state-action value function Q
while termination condition not met **do**
    Execute the behavior policy to obtain a new experience (s, a, s', r)
    Perform TD update for Q using the new experience

$$Q(s, a) \leftarrow Q(s, a) + \alpha(r + \gamma \max_{a'} Q(s', a') - Q(s, a)).$$

- Various termination criteria can be used
  - e.g. little change over recent updates, maximum number of interaction, maximum computation time
- A commonly used behavior policy is the ϵ-greedy policy, which executes a random action w.p. $\epsilon > 0$, and the greedy action $\text{argmax}_a Q(s, a)$ w.p. $1 - \epsilon$.

MDPs with a very large state space – function approximation

- If we have too many states, we can't use a table to store the Q-function.
- Typically, we use a parametric representation $Q_\theta(s, a)$ in this case.
- The update step in the Q-learning algorithm becomes

$$\theta \leftarrow \theta - \alpha(Q_\theta(s, a) - r - \gamma \max_{a'} Q_\theta(s', a')) \nabla Q_\theta(s, a).$$

  Why? This performs a gradient descent on the squared TD error

$$(Q_\theta(s, a) - r - \gamma \max_{a'} Q_{\theta^-}(s', a'))^2,$$

  where $\theta^- = \theta$ is treated as fixed parameters.
- Tabular Q-learning is a special case: $Q_\theta(s, a) = \sum_{s' \in S} \theta_{s'} I(s' = s)$.

Algorithm Q-learning with function approximation

```
Initialise the state-action value function Q_θ
while termination condition not met do
    Execute an appropriate behavior policy to obtain a new experience (s, a, s', r)
    Perform TD update
```

$$\theta \leftarrow \theta - \alpha(Q_\theta(s,a) - r - \gamma \max_{a'} Q_\theta(s',a'))\,\nabla Q_\theta(s,a).$$

SARSA

- SARSA is the same as Q-learning, except that for each update, it first observes a sequence s, a, r, s', a' (that's why the name SARSA), then update

$$Q(s, a) \leftarrow Q(s, a) + \alpha(r + \gamma Q(s', a') - Q(s, a)).$$

- Function approximation can be applied too.

$$\text{Q-learning (off-policy):} \quad Q(s,a) \leftarrow Q(s,a) + \alpha(\overbrace{r + \gamma \max_{a'} Q(s',a')}^{\text{target is greedy policy}} - Q(s,a))$$

$$\text{SARSA (on-policy):} \quad Q(s,a) \leftarrow Q(s,a) + \alpha(\overbrace{r + \gamma Q(s',a')}^{\text{target is } \epsilon\text{-greedy policy}} - Q(s,a)).$$

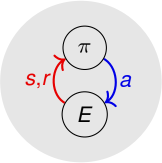
- Q-learning is off-policy as target policy (greedy) $\neq$ behavior policy (ϵ-greedy).
- SARSA is an on-policy as target policy = behavior policy.

environment model

bandits, MDPs, POMDPs

learning target

model, value, policy



behavior policy

ϵ*-greedy*

update rules

TD minimization

Q-learning/SARSA in the big picture

Policy Optimization Methods

- Why policy optimization?
  - learn Q: computing the policy $\pi(s) = \operatorname{argmax}_a Q(s, a)$ can be hard
  - learn V: requires lookahead and optimize
  - learn π: policy is directly available
- Various policy optimization algorithms: REINFORCE, actor-critic, DPG, ...

REINFORCE

- REINFORCE (Williams, 1992) directly optimises a parametric policy $\pi_\theta(a \mid s)$ by maximizing its value function

$$V(\theta) = \sum_{\tau} p(\tau \mid \theta) R(\tau) = \mathbb{E}_{\tau \sim p} R(\tau),$$

where

  - $\tau = (s_0, a_0, s_1, a_1, \ldots)$ is a trajectory (state-action sequence),
  - $p(\tau \mid \theta)$ is the distribution of trajectory τ when playing π_θ, and
  - $R(\tau)$ is the total (discounted) reward collected along τ.

- It computes a stochastic gradient of $V(\theta)$ at each iteration, and then performs gradient ascent.
- A simple parametrization: $\pi_\theta(a \mid s)$ as a logistic regression model.

- Usually, it is often computationally intractable to evaluate $V(\theta)$ first, and then evaluate its gradient,
  - in the discrete state case, $V(\theta)$ involves summing over a large number of trajectories.
  - in the continuous state case, computing $V(\theta)$ involves evaluating a complex integral.

Policy gradient theorem

$$\nabla V(\theta) = \mathbb{E}_{\tau \sim p} R(\tau) \nabla \ln p(\tau \mid \theta).$$

- Why? Because $\nabla \ln p(\tau \mid \theta) = \frac{\nabla p(\tau \mid \theta)}{p(\tau \mid \theta)}$.
- This gives us a Monte Carlo estimate of the gradient

$$\nabla V(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} R(\tau^{(i)}) \nabla \ln p(\tau^{(i)} \mid \theta),$$

  where the trajectories $\tau^{(1)}, \ldots, \tau^{(N)}$ are randomly sampled from $p(\cdot \mid \theta)$.
- We need to relate $\nabla \ln p(\tau \mid \theta)$ back to the policy π_θ.

Policy gradient theorem

$$\nabla V(\theta) = \mathbb{E}_{\tau \sim p} \sum_{t=0}^{|\tau|-1} R(\tau) \nabla \ln \pi_\theta(a_t \mid s_t).$$

where $|\tau|$ denotes the length of a trajectory (number of state-action pairs).

policy gradient learning = weighted log-likelihood maximization

- Why? Note that

$$p(\tau \mid \theta) = p(s_1) \prod_{t=0}^{|\tau|-1} \pi(a_t \mid s_t, \theta) p(s_{t+1} \mid s_t, a_t)$$

$$\nabla \ln p(\tau^{(i)} \mid \theta) = \sum_{t=0}^{|\tau_i|-1} \nabla \ln \pi_\theta(a_t^{(i)} \mid s_t^{(i)}).$$

- While $p(\tau^{(i)} \mid \theta)$ depends on the transition probabilities, the gradient of the log probability does not!

Algorithm REINFORCE algorithm

while not terminated **do**
    Simulate π_θ to collect trajectories $\tau^{(1)}, \ldots, \tau^{(N)}$.
    Update θ using

$$\theta \leftarrow \theta + \alpha \left(\frac{1}{N} \sum_{i=1}^{N} \left(\sum_{t=0}^{|\tau^{(i)}|-1} R(\tau^{(i)}) \nabla \ln \pi_\theta(a_t^{(i)} \mid s_t^{(i)}) \right) \right).$$

REINFORCE is an on-policy method.

Policy gradient theorem

$$\nabla V(\theta) = \mathbb{E}_{\tau \sim p} \sum_{t=0}^{|\tau|-1} \gamma^t (R(\tau_{\geq t}) - b(s_t)) \nabla \ln \pi_\theta(a_t \mid s_t).$$

where $\tau_{\geq t} = (s_t, a_t, \ldots)$ for $\tau = (s_1, a_1, \ldots)$, and $b(s)$ is an arbitrary function of state.

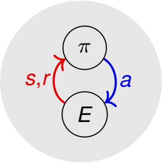
- Focus on future but not past: $\tau_{<t} = (s_0, a_0, \ldots, s_{t-1}, a_{t-1})$ has no effect on $\pi_\theta(a_t \mid s_t)$.
- Use a baseline $b(s)$ for variance reduction.
- More policy gradients: (Schulman et al., 2015)

environment model

bandits, MDPs, POMDPs

learning target

model, value, policy



behavior policy

target policy

update rules

weighted likelihood maximization

policy optimization in the big picture

Model-based RL

Algorithm A general model-based RL approach

1: initialize an estimated environment model $\tilde{M}$
2: **for** $t = 1, 2, \ldots$ **do**
3: compute optimal policy $\tilde{\pi}^*$ for $\tilde{M}$
4: collect experience by running the ϵ-greedy policy $\tilde{\pi}^*_\epsilon$
5: update the environment model $\tilde{M}$ based on collected experience

- $\tilde{\pi}^*_\epsilon$ folows $\tilde{\pi}^*$ with probability $1 - \epsilon$ and takes a random action otherwise.

Tabular model-based RL

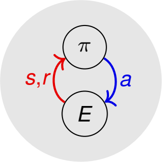
- Initialization
  - Each $R(s, a)$ can be initialized to the maximum possible value to encourage exploration
  - $\tilde{M}$ can be initialized with a "pseudo-count" $n_{s,a,s'}$ for each transition (s, a, s').
- Update
  - Reward update: compute average of rewards encountered.
  - Transition model update: update the transition count $n_{s,a,s'}$ to include both the pseudo-count and the actual count, then compute $T(s'|s, a) = n_{s,a,s'} / \sum_{s''} n_{s,a,s''}$.
- The planning problem of computing $\tilde{\pi}^*$ for $\tilde{M}$ can be solved using value/Q iteration.

environment model

bandits, MDPs, POMDPs

learning target

model, value, policy



behavior policy

ϵ*-greedy*

update rules

reward regression, transition density estimation

simple model-based RL in the big picture

- computing average reward is least squares regression
- frequency-based transition probability is regularized maximum likelihood estimation

- Advanced model-based RL: PlaNet (Hafner et al., 2019b), DreamerV1 (Hafner et al., 2019a), DreamerV2 (Hafner et al., 2020), DreamerV3 (Hafner et al., 2023)
- Model-based methods can be more sample efficient than model-free methods.

Roadmap

- Introduction and overview

  motivation, bandits, big picture
- Classical ideas

  temporal difference methods, policy gradient, ...
- Deep Reinforcement learning

  neural networks, DQN, DDPG, ...
- Advanced techniques

  representation learning, stabilization, few-shot learning
- Applications

  AlphaGo, AlphaTensor, ...

References I

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